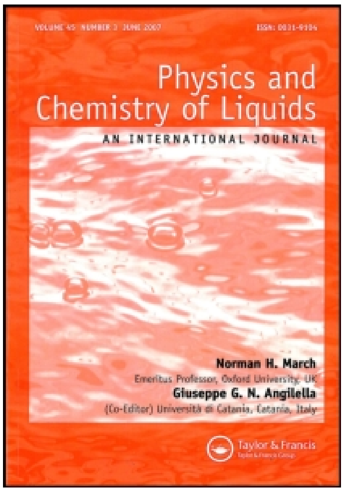

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Thermodynamics of the Equilibrium Between a Fractional Quantum Hall liquid and a Wigner Electron Solid

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Letter

THERMODYNAMICS OF THE EQUILIBRIUM BETWEEN A FRACTIONAL QUANTUM HALL LIQUID AND A WIGNER ELECTRON SOLID

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A recent paper of Wu *et al.*, discusses thermodynamic observables in a fractional quantum Hall (FQH) liquid. In particular the de Haas-van Alphen effect is considered theoretically. Here, it is pointed out that laboratory experiments on GaAs/AℓGaAs heterojunctions, plus the analogue of the Clausius–Clapeyron equation in an applied magnetic field, allow schematic analysis of the orbital magnetism of the FQH liquid. There is general agreement with the theoretical results of Wu *et al.*

Keywords: Quantum hall effect; orbital magnetism; Wigner solid

In a recent article, Wu *et al.* [1] have given a very detailed theoretical treatment of the thermal activation of quasiparticles and the thermodynamic observables in fractional quantum Hall (FQH) liquids. Their important conclusion is that what are usually thought of as different but equivalent pictures of the FQH effect (anyon, composite Fermion, and composite Boson) can exhibit significant differences at finite

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temperature, though they are all equivalent at $T = 0$. Then their final sentence reads 'In particular, it is more desirable that these theoretical predictions would be put to experimental tests, if the tremendous difficulties in measuring thermodynamic quantities of a thin layer of electron gas could be overcome someday'.

It is therefore the more remarkable that some existing experiments on GaAs/AℓGaAs heterojunctions in magnetic fields **B** applied perpendicular to the two-dimensional electron assembly can provide insight into some of the thermodynamic properties of the FQH liquid studied theoretically by Wu *et al.* [1]. The two laboratory experiments focussed on below are those of Andrei *et al.* [2] and the subsequent, quite different, type of investigation by Buhmann *et al.* [3]. Both experiments, in spite of their very different techniques, allow information to be extracted on the equilibrium between a FQH liquid and a Wigner electron solid, in which electron localization driven purely by Coulomb interaction in Wigner's original papers [4, 5] is now magnetically induced [6]. We wish to add to these two laboratory experiments the quantum computer simulation results of Ortiz, Ceperley and Martin [7] (OCM).

Starting with the computer simulation data, OCM for $r_s = 20$, r_s as usual measuring the mean interelectronic spacing, have by means of a stochastic method appropriate for systems with broken time-reversal symmetry, studied the transition between an incompressible $\nu = 1/m$ FQH liquid and a Wigner solid. For $m = 5$, the work of OCM demonstrates that 'further work must be done to show definitively which phase is stable'.

Returning to the laboratory experiments, Buhmann *et al.*, as a result of their measured luminescence spectra, have proposed a qualitative form of the phase diagram showing 4 Wigner solid phases, with the FQH liquid assumed as the ground-state at filling factors $\nu = 1/5$, $1/7$ and $1/9$ (see their Fig. 4(c)). The earlier radio spectroscopic data of Andrei *et al.* [2] are generally compatible with the Buhmann *et al.*, phase diagram, though the features shown in this diagram differ most from Andrei *et al.*, around $\nu = 1/5$, $1/7$ and $1/9$. However, the computer simulation results are not incompatible with the Buhmann *et al.* (T, ν) phase diagram in their Figure 4(c).

We now turn to the thermodynamic interpretation [8] of the Buhmann *et al.*, phase diagram, in relation to the calculation of thermodynamic observables, and in particular the de Haas-van Alphen effect, by Wu *et al.* [1]. Lea, March and Sung [8] use, in particular, to discuss the thermodynamics of an electron solid to electron liquid first-order melting transition, the result for the melting temperature T_m as a function of magnetic field H, at constant area Ω:

$$\left(\frac{\partial T_m}{\partial H}\right)_\Omega = -\frac{\Delta M}{\Delta S}. \tag{1}$$

If the subscript s denotes the solid phase and ℓ the liquid phase, then ΔM in Eq. (1) is $M_\ell - M_s$ which is the change in magnetization on melting, while $\Delta S = S_l - S_s$ is the corresponding entropy change. Using the Landau level filling factor ν, given in terms of the (areal) electron density n and the magnetic field H applied perpendicular to the electron layer in the heterojunction:

$$\nu = \frac{nhc}{eH}, \tag{2}$$

one immediately recasts Eq. (1) into a relation used for interpreting the Buhmann *et al.* (T, ν) phase diagram, namely

$$\left(\frac{\partial T_m}{\partial \nu}\right)_\Omega = \left(\frac{H}{\nu}\right)\frac{\Delta M}{\Delta S}. \tag{3}$$

Using this Eq. (3), and making plausible assumptions about the entropy change on melting, Lea *et al.* [8] in their Figure 3 draw a schematic diagram of the change in magnetization ΔM on melting along the melting curve of the Buhmann *et al.*, proposed phase diagram. Lea *et al.*, conclude that ‘this field dependence of ΔM is very reminiscent of the de Haas-van Alphen effect at integral ν values, suggesting that the magnetism of the electron liquid phase is intimately connected with the exotic variation of ΔM ...’. Though the range of ν values in Figure 3 of Lea *et al.*, is somewhat different from that in the very recent discussion of Wu *et al.* [1], the gist of the conclusions is the same in the two cases.

In later work, Lea *et al.* [9] use both anyon and composite Fermion models to represent the main features of the melting of such Wigner electron solids as a function of the Landau level filling factor. Of course, the later studies of Wu *et al.*, transcend in their detailed treatment the results in references 8 and 9.

In summary, the detailed theoretical study of Wu *et al.* [1] of thermodynamic observables in FQH liquids makes it now the more urgent to bring such theoretical predictions into quantitative contact with (a) laboratory experiments such as reported in Refs. [2, 3] and (b) quantal computer simulations of the kind discussed in Ref. [7]. In the latter area, it would seem of great interest to study the orbital magnetism of the FQH liquid in the range of ν values discussed in Refs. [8, 9].

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References

[1] Wu, Y. S., Yu, Y., Hatsugai, Y. and Kohmoto, M. (1998). *Phys. Rev.*, **B57**, 9907.
[2] Andrei, E. Y., Deville, G., Glattli, D. C., Williams, F. I. B., Paris, E. and Etienne, B. (1988). *Phys. Rev. Lett.*, **60**, 2765.
[3] Buhmann, H., Joss, W., von Klitzing, K., Kukuskim, I. V., Plant, A. S., Martinez, G., Ploog, K. and Timofeev, V. B. (1991). *Phys. Rev Lett.*, **B6**, 926.
[4] Wigner, E. P. (1934). *Phys. Rev.*, **46**, 1002.
[5] Wigner, E. P. (1938). *Trans. Faraday Soc.*, **34**, 678.
[6] Durkan, J., Elliott, R. J. and March, N. H. (1968). *Rev. Mod. Phys.*, **40**, 812.
[7] Ortiz, G., Ceperley, D. M. and Martin, R. M. (1993). *Phys. Rev. Lett.*, **71**, 2777.
[8] Lea, M. J., March, N. H. and Sung, W. (1991). *J. Phys. Condens. Matter*, **3**, CL4301.
[9] Lea, M. J., March, N. H. and Sung, W. (1992). *J. Phys. Condens. Matter*, **4**, 5263.